MA2160
Test 2
Spring 2007

Name:________________________________

Instructions: **You may use your calculator on the entire test. You must show enough work to justify all answers.**

Page	Score	
1		/ 14
2		/ 20
3		/ 15
4		/ 20
5		/ 15
6		/ 16
Final score		/ 100

1. The approximation to a definite integral using $n = 20$ is 3.456; the exact value is 5. If the approximation was found using each of the following rules, use the same rule to estimate the integral with $n = 60$.

   (a) MID

   Solution.

$$\text{Error} = 5 - 3.456 = 1.544.$$
$$\frac{1.544}{9} = .171556$$
$$5 - .171556 = 4.82844$$

□

4.82844 [7]

   (b) SIMP

   Solution.

$$\text{Error} = 5 - 3.456 = 1.544.$$
$$\frac{1.544}{81} = .019062$$
$$5 - .019062 = 4.98094$$

□

4.98094 [7]

2. Compute the integral, if it converges, or show that it diverges. You must use limits properly for full credit.

   (a) $\int_1^6 \frac{1}{x-4}\, dx$ [10]

   Solution.

$$\int_1^6 \frac{1}{x-4}\, dx = \int_1^4 \frac{1}{x-4}\, dx + \int_4^6 \frac{1}{x-4}\, dx = \lim_{a\to 4^-} \int_1^a \frac{1}{x-4}\, dx + \lim_{b\to 4^+} \int_b^6 \frac{1}{x-4}\, dx$$

$$= \lim_{a\to 4^-} \ln|x-4|\Big|_1^a + \lim_{b\to 4^+} \ln|x-4|\Big|_b^6 = \lim_{a\to 4^-} \Big(\ln|a-4| - \ln 3\Big) + \lim_{b\to 4^+} \Big(\ln 2 - \ln|b-4|\Big)$$

$$= -\infty - \ln 3 + \ln 2 + \infty.$$

   Therefore, the integral diverges.
   [Note: $\infty - \infty \neq 0$.] $\square$

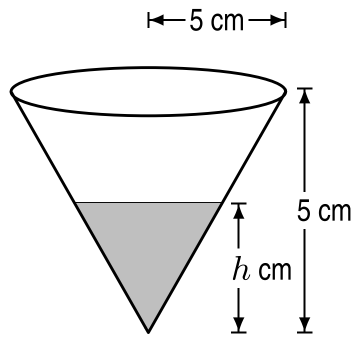
   (b) $\int_0^\infty \frac{2x}{e^x}\, dx$ [10]

   Solution.

$$\int_0^\infty \frac{2x}{e^x}\, dx = \lim_{a\to\infty} \int_0^a \frac{2x}{e^x}\, dx = \lim_{a\to\infty} \frac{-2x}{e^x} - \frac{2}{e^x}\Bigg|_0^a = \lim_{a\to\infty} \frac{-2a}{e^a} - \frac{-2}{e^a} + 2 = 2$$

$\square$

3. Consider the cone below.



(a) Write a Riemann sum representing the volume of the cone. [10]

Solution.

$\frac{w}{10} = \frac{h}{5}$ or $w = 2h$. That is, $r = h$.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \pi(h_i)^2 \Delta h$$

□

(b) Write a definite integral representing the volume of the cone. You do not need to evaluate the integral. [5]

Solution.

$$\pi \int_0^5 h^2 \, dh$$

□

4. Consider the region bounded by $y = x^2$, $y = x$, $x = 0$, and $x = 1$.

   (a) Find a definite integral representing the volume of the solid formed by rotating this region around the line $x = -2$. [15]

   Solution.

$$\pi \int_0^1 (\sqrt{y} + 2)^2 - (y + 2)^2 \, dy$$

$\square$

   (b) Evaluate the integral to find the volume of this solid. [5]

   Solution.

$$\pi \int_0^1 (\sqrt{y} + 2)^2 - (y + 2)^2 \, dy = \pi \int_0^1 \left(-y^2 - 3y + 4\sqrt{y} \right) dy = \pi \left(-\frac{y^3}{3} - \frac{3y^2}{2} + \frac{8y^{3/2}}{3} \Bigg|_0^1 \right) = \frac{5\pi}{6}$$

$\square$

5. When an oil well burns, sediment is carried up into the air by the flames and is eventually deposited on the ground within 100 miles of the well. Less sediment is deposited further away from the oil well. Experimental evidence indicates that the density (in tons/square mile) at a distance r from the burning oil well is given by

$$\delta(r) = \frac{7}{1+r^2}.$$

Find and evaluate an integral which represents the total mass in the deposit. Specify the units. [15]

Solution.

$$2\pi \int_0^{100} \frac{7r}{1+r^2}\, dr = 7\pi \ln(10001) \approx 202.548 \text{ tons}$$

□

6. A rectangular swimming pool 50 ft long, 20 ft wide, and 10 ft deep is filled with water to a depth of 9 ft. Write and evaluate an integral to find the work required to pump all of the water out over the top. Specify the units. [16]

Solution.

$$62.4\int_0^9 1000(10-h)\,dh = 62400\int_0^9 (10-h)\,dh = 62400\left(10h - \frac{h^2}{2}\Bigg|_0^9\right) = 3,088,800 \text{ ft-lbs.}$$

□